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A REVIEW OF SEPARATED AND REATTACHING FLOWS WITH HEAT TRANSFER

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Abstract—Theoretical and experimental aspects of separated and reattaching flows are surveyed with special reference to the heat-transfer characteristics of such flows.

NOMENCLATURE

M,	Mach number;
Re,	Reynolds number;
Pr,	Prandtl number;
St,	Stanton number (average value);
U,	free-stream velocity;
u,	local velocity in mixing or shear layer;
p,	static pressure;
Δp,	pressure difference relative to free stream;
ρ,	density;
μ,	viscosity;
T,	temperature;
C_p,	specific heat at constant pressure;
$\underline{C}p$,	pressure coefficient;
$\bar{h}$,	average heat-transfer coefficient;
x,	distance from leading edge;
y,	distance normal to surface;
h,	step, spoiler or cavity height;
L,	step or body length.
Q,	rate of heat flow.

Subscripts

a,	adiabatic;
d,	dead-air region;
e,	outer edge of mixing or shear layer;
o,	conditions at beginning of interaction;
s,	separation;
A,	attached flow;
W,	wall.

1. INTRODUCTION

THE PHENOMENON of flow separation is encountered throughout the field of fluid dynamics [1–4] and often limits the usefulness and efficiency of aerodynamic devices; typical examples are the stall of an aerofoil, which increases drag and reduces lift, and the separation of internal flows through fans, engine intakes and wind-tunnel diffusers. Controlled separation, however, may be advantageous—as in the case of slender delta wings, spoilers used on wings for control purposes and transverse fins used to improve the heat-transfer performance of nuclear reactor fuel elements. In high-speed flow the problem of separation involves the effects of compressibility, shock-wave boundary-layer interaction and aerodynamic heating. Since separation of flow may be used as a means of controlling the heat transfer to a surface and for aerodynamic control purposes, the heat-transfer characteristics of separated flows are of considerable interest in the design of heat exchangers, aircraft and space vehicles. Practical applications require a knowledge of the effects of Mach number, Reynolds number, wall to free-stream temperature ratio and wall geometry, especially curvature, on the pressure-distribution and heat-transfer characteristics of the separated and reattaching boundary layer.

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In order to predict and prevent or delay flow separation, a great deal of theoretical and experimental research effort has been expended. Although this effort, mainly on simple types of flow, has led to a partial understanding of the physical nature of the flow mechanism, the large number of variables involved makes satisfactory theoretical solutions and generalization of experimental results difficult to achieve. The object of the survey is therefore to review previous work on separated flows with special emphasis on those features which are still in question and require further investigation to provide a better understanding of separated flow with heat transfer.

2. COMPARISON OF SUBSONIC AND SUPERSONIC SEPARATION

Lighthill [5] has described separation of subsonic and supersonic flow in relation to the upstream influence of disturbances through the boundary layer. In supersonic flow, a disturbance leading to a positive pressure gradient causes the boundary layer to thicken and it must then begin to curve slightly upstream of the disturbance. Due to the relation between pressure gradient and streamline curvature in supersonic flow [3], the curvature produces a positive pressure gradient slightly upstream. This in turn produces more thickening and the process repeats itself with the pressure gradient gradually decaying upstream. The mechanism is also applicable to a negative pressure gradient, which causes the boundary layer to thin, and is peculiar to supersonic flow. If a compressive disturbance is sufficiently large, e.g. a strong shock wave, it causes separation at the surface and in the modified external flow the pressure begins to rise ahead of the separation point. The separation point then moves upstream until the wedge-shaped reversed-flow or "dead-air" region is slender enough to cause no further separation ahead of it. The dead-air pressure is then related to the wedge angle by the oblique-shock flow deflection equations [3]. This separation mechanism is supplementary to the pressure-gradient mechanism in supersonic flow but has a much greater upstream influence by virtue of the reversed-flow region, it is also typical of subsonic flow, e.g. flow separation behind a circular cylinder and in both flows the upstream influence is greater for laminar layers, since these are more readily separated than turbulent layers.

The subsonic flow up a step has been analysed by Lighthill and is compared with the corres-

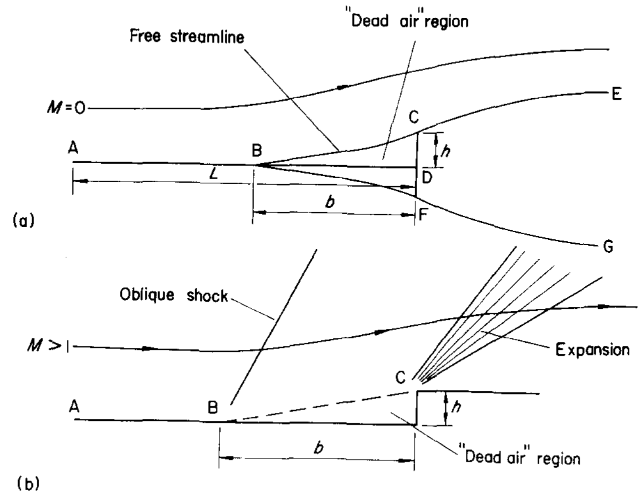


FIG. 1. Flow up a step: (a) Subsonic flow; (b) Supersonic flow.

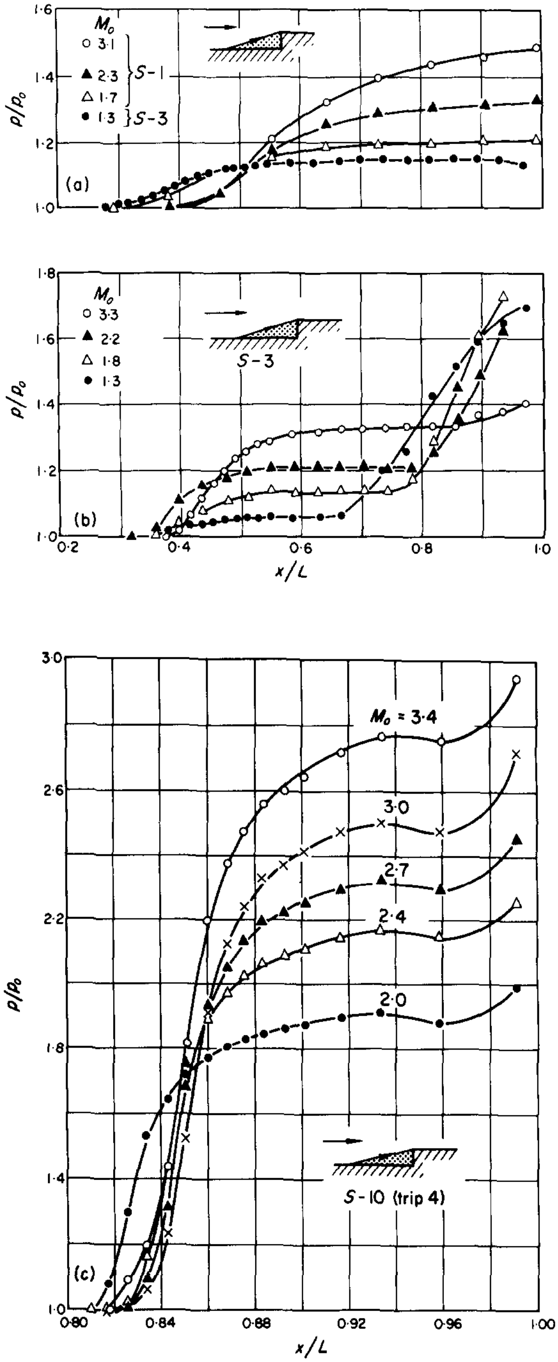


FIG. 2. Effect of Mach number on step flow pressure distribution for laminar, transitional and turbulent separation [7]: (a) Pure laminar separation, $Re_L \approx 0{\cdot}13 \times 10^6$; (b) Transitional separation, $Re_L \approx 0{\cdot}60 \times 10^6$; (c) Turbulent separation; $Re_L \approx 2{\cdot}6 \times 10^6$.

ponding supersonic flow in Fig. 1. For subsonic step flow the position of separation B, specified by b/h and the dead-air pressure coefficient Cp_d $(= \Delta p/\frac{1}{2}\rho U^2)$ on the free streamline BC, depend on the length–height ratio L/h of the step and critically on the state of the boundary layer (laminar or turbulent). For supersonic step flow the free streamline is much more nearly straight and the wedge-shaped dead-air region adjusts itself so that the pressure increase is just sufficient to cause the boundary layer to separate, the values of b/h and Cp_d being obtained from the oblique-shock flow deflection relation [3].

Due to the pressure-gradient mechanism of upstream influence causing thickening of the boundary layer ahead of the separation point, the rise in pressure across a shock does not occur abruptly at the surface. Instead, the pressure tends to rise exponentially at first and then more uniformly to the separation pressure, after which some further rise continues to the "plateau pressure" of the dead-air region, Fig. 2, the distances involved being of the order of ten boundary-layer thicknesses for a laminar layer and one for a turbulent layer. For given upstream conditions Bogdonoff and Kepler [6] have shown that the surface pressure distribution up to and including separation is the same for both step and incident-shock induced separation. This independence of the mode of separation is a simplifying feature of well-separated supersonic flow described by Chapman *et al.* [7] as "free interaction". Typical supersonic interactions, e.g. corner flow, incident-shock and normal-shock flows, are shown in Fig. 3. For comparison with subsonic separation, free interaction separation pressure coefficients at $M = 2$ are $Cp_s \doteqdot 0{\cdot}03$ for a laminar layer at $Re_x = 5 \times 10^5$ and $Cp_s \doteqdot 0{\cdot}23$ for a turbulent layer. Lighthill concludes that the pressure rise inducing separation is much smaller in supersonic flow due to the inability of the main stream flow to transmit the sudden pressure rise upstream independently of the boundary layer as happens in the low-speed case.

Chapman's analysis [7] for the simplified

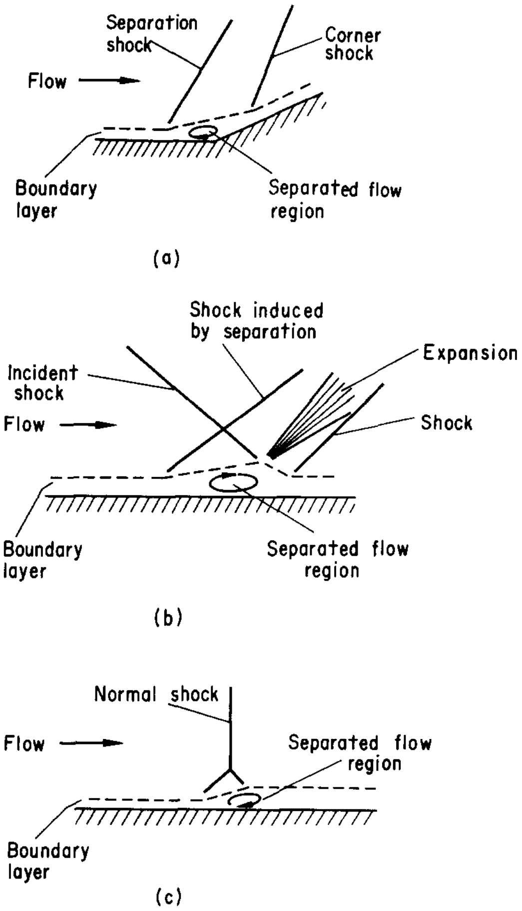


FIG. 3. Supersonic interactions: (a) Corner flow; (b) Incident shock flow; (c) Normal shock.

case of pure laminar leading-edge separation predicts that Cp_d decreases with increase of M, as shown below.

M	0	1·5	2	3	4
Cp_d	0·526	0·35	0·25	0·14	0·09

The value of $Cp_d = 0{\cdot}526$ for incompressible laminar separation agrees well with experimental results for flows with negligible boundary-layer thickness at separation, e.g. circular cylinder. The values of Cp_d for supersonic separation are also in fair agreement with experiment.

3. SUPERSONIC SEPARATED FLOW

3.1 *General investigations*

A good deal of experimental information on interactions between shock waves and boundary layers has become available since Ackeret *et al.* [8] first investigated the interaction of near normal shocks and curved surfaces encountered in transonic flow past aerofoils. Holder *et al.* [9] reviewed the literature up to 1954 when the extension of fundamental results to practical applications was again considered mainly in relation to the performance of aerofoils with turbulent boundary layers at transonic speeds. The effect of Reynolds number or scale effect of shock wave influence on laminar layers was shown to be a source of discrepancy between model and full-scale results unless transition is fixed. It was also pointed out that laminar-flow results are applicable only to low Reynolds number conditions, e.g. propeller and compressor blades and high-speed high-altitude flight. Results from simplified two-dimensional separations showed the dependence of the flow on Mach number, Reynolds number and incident shock strength in laminar, transitional and turbulent interactions. The correlation of separation, transition and reattachment with the wall pressure distribution was shown in detail.

Information provided by Gadd [10] on separation conditions is quite useful since the separation pressure is often related to the nearly constant plateau pressure. Data from free interactions also provide a design guide for the avoidance of separation effects and for Mach numbers in the range 1·1–3·5 the separation coefficient for free laminar interactions may be taken as $0{\cdot}94\,(M^2-1)^{-\frac{1}{4}}\,Re_{xs}^{-\frac{1}{4}}$ (Bray *et al.* [11]) in the Reynolds number range 10^4–10^6 and for free turbulent interactions to decrease as $(M^2-1)^{-\frac{1}{4}}$ approximately, (Schuh [12]), virtually independent of Reynolds number in the range 10^6–2×10^7 (Love [13]). The pressure coefficient associated with laminar separation is seen to be much smaller than that for turbulent separation, in which there is a greater variation in pressure across the boundary layer making turbulent separation less amenable to theoretical treatment. In general heat-transfer and aerofoil surface curvature effects are small, although Gadd [10] emphasises that the theories do not cope satisfactorily with heat-transfer effects.

The investigation of Chapman *et al.* [7] is typical of those in which variable-density wind tunnels have been used to vary Mach and Reynolds numbers independently. In this case, a variety of separated laminar, transitional and turbulent flows, e.g. incident shock, step and corner types, were studied at Mach numbers in the range 0·4–3·6 and Reynolds numbers in the range 4×10^3–5×10^6. Typical pressure distributions are those for supersonic step flow, shown in Fig. 2. The small pressure dip in the turbulent flow pressure distribution is associated by Bogdonoff and Kepler [6] with a strong vortex located in the supposedly dead-air region, in which at $M = 3{\cdot}0$ flows up to $M = 0{\cdot}3$ were found.

Further experimental data on turbulent separated flows with heat transfer, involving steps. spoilers and cavities, have been obtained by Gadd [14], Gadd *et al.* [15], Larson [16], Thomman [17] and Charwat *et al.* [18]. Zero heat-transfer spoiler pressure distributions in turbulent flow are given by Mueller [19] at $M = 1{\cdot}93$ and by Heyser and Maurer [20] at $M = 0{\cdot}6$–$2{\cdot}8$, who also report the effect of jet spoilers.

3.2 *Effect of wall temperature on transition*

When laminar separation occurs the importance of transition to turbulence before reattachment of the separated layer, and the predominance of such transitional separations, have been emphasized by Chapman *et al.* [7], who found that such separations are generally unsteady and the pressure distribution relatively strongly influenced by Reynolds number. It was also found that the stability of a separated laminar layer increases significantly with Mach number, compared with an attached flat plate layer under similar conditions of constant pressure, free-stream turbulence and zero heat transfer. Typically, increasing the Mach number raises the separated-layer transition Reynolds number from 5×10^4 at $M = 0{\cdot}3$ to 4×10^5 at $M = 3{\cdot}0$. This favourable effect makes separated laminar boundary layers of practical interest at hypersonic Mach numbers. However, the effect of wall cooling ($T_W/T_{W_a} < 1$) is required, since in practical applications aerodynamic heating at high Mach numbers is usually associated with wall temperatures much lower than the adiabatic temperature. Larson [16] found that in all cases the transition data for values of $T_W/T_{W_a} = 0{\cdot}4$–1 exhibit a favourable effect of Mach number, but that the effect of wall cooling at constant Mach number has an adverse effect on transition Reynolds number. This destabilizing effect is opposite to the stabilizing effect on attached boundary layers observed for moderate wall cooling, Schlichting [21].

distribution

3.3.1 *Stratford–Gadd method.* Gadd [22] has investigated theoretically the effects of Mach number, Reynolds number, wall temperature and surface curvature on two-dimensional laminar separation by an extension of Stratford's method for incompressible flow. In this method the boundary layer is divided into an outer region, shown to be essentially inviscid when the adverse pressure gradients are fairly sharp, so that the outer part of the mass-flow profile is determined from inviscid flow considerations. The inner profile is then determined by the conditions that it must join smoothly onto the outer profile, that continuity must be satisfied and that at the wall the rate of change of viscous stress must balance the pressure gradient. In the supersonic case, the pressure is related to the external flow deflection caused by the thickening of the boundary layer. The analysis gives the result that the pressure coefficient at separation is unaffected by wall temperature, but predicts that the pressure gradient at separation varies as $T_W^{-\frac{3}{2}}$. Convex surface curvature, typical of a two-dimensional aerofoil, reduces the pressure coefficient at separation.

Before making a general experimental investigation of heat-transfer effects, Gadd [23] compared the predictions of theories based on arbitrarily fixed pressure distributions. Cases with linear adverse velocity gradient, velocity profiles of constant shape and zero skin friction, and an abrupt pressure gradient provoking separation without any pressure increase, according to a Polhausen-type analysis, were considered. These theories predicted that for laminar layers $Cp_s \propto (T_W/T_{W_a})^{-n}$, where $n = 0{\cdot}5$–$1{\cdot}0$, which results agree qualitatively with the Stratford–Gadd method in that cooling or heating the wall makes the boundary layer more or less difficult to separate.

Gadd's experimental results [14] for laminar layers at $M = 3$, show that there may be an effect of wall temperature on Cp_s, although the pressure distribution is virtually unaffected by a decrease of an eighth and an increase of a quarter in absolute temperature. Similar temperature differences produced observable trends in the separated turbulent layer pressure distribution, the upstream effect increasing when the wall is heated and decreasing when it is cooled. No theory was developed for the effects of heat transfer on turbulent separation but it was suggested that the effect is associated with the influence of wall temperature on the boundary-layer displacement thickness; heating the wall increases the displacement thickness and reduces the velocities in the boundary-layer.

Gadd concludes that in most cases with moderate rates of heat transfer, the zero heat-transfer pressure distribution is applicable. This result is particularly useful, since in practical supersonic separations the wall temperature is generally lower than the adiabatic temperature. This independence also enables wind-tunnel pressure-distribution observations to be made before model and flow are in thermal equilibrium.

3.3.2 *Crocco–Lees method.* Crocco and Lees [24] have approached the problem of the characteristic interaction between an external isentropic flow and an internal dissipative flow, which occurs in shock-wave boundary-layer interaction with flow separation, by analysing a simplified theoretical model in which mixing is considered as the fundamental physical process determining the pressure rise. This "mixing theory" is an attempt to eliminate the shortcomings of the Kárman–Pohlhausen momentum integral method, in which the velocity gradient is uniquely determined by a single pressure-gradient parameter, the main objection being that in the constant pressure plateau region the velocity profiles are very different from those on a flat plate with zero pressure gradient.

The external flow is taken to be a plane steady supersonic flow which makes a small angle with a plane surface. The internal flow is regarded as almost one-dimensional with appropriately defined mean velocity and temperature. The profiles are assumed not to vary in shape with distance along the wall, so that the Falkner–Skan similar flow solutions may be used. For these solutions the velocity is positive everywhere along the upper branch, while regions of reversed flow exist along the lower branch, the two branches joining smoothly at the separation or reattachment point. The pressure distribution in compressible flow is found by use of the Stewartson transformation, which may be applied to laminar flow but is of doubtful validity for turbulent flow. By integrating the equations of continuity and momentum across the boundary layer, the deflection of the external flow induced by the boundary-layer growth is related to the pressure distribution through the local Prandtl–Meyer relation, the equations involving parameters based on the velocity and temperature profiles. To enable a solution to be obtained, a semi-empirical mixing coefficient is introduced, and for separated flows the skin-friction coefficient is assumed zero. The mixing coefficient is based on generalized experimental results, the value for turbulent mixing being of the order of ten times the laminar value.

The original theory is limited to zero heat transfer and constant static pressure normal to the surface, which assumption is questionable for turbulent separation. Vasiliu [25], however, has calculated the pressure distribution in regions of step induced turbulent separation, assuming that the mixing coefficient varies linearly between constant values in the separation and reattachment zones. The agreement with experimental results at Mach numbers of about 3 and 4 is good, although the pressure dip just upstream of the reattachment pressure rise is not predicted by the theory. Crocco [26] has since dealt with the extension of the Stewartson transformation to turbulent flows, concluding that for a streamwise pressure gradient the results are invalid. Two alternative transformations which cover the general case of heat-transfer and pressure gradient are given.

In order to eliminate the semi-empirical features of the Crocco–Lees method and include the effect of heat transfer, several modifications have been made to the original mixing theory. Bray *et al.* [11] have compared the Crocco–Lees method with other methods for laminar shock-wave boundary-layer interactions involving separation, heat transfer and suction. Instead of the original Crocco–Lees parameters governing the form of the velocity profile, a lower branch similar solution of Cohen and Reshotko, was used, which more closely resembles the reversed flow velocity profile near the wall. It was found that the computed results were qualitatively more in accord with experiment than those previously obtained by Bray at $M = 2$, in spite of an unrealistic negative pressure gradient at the plateau. Lees and Reeves [27] have extended an approximate method of Tani for attached laminar layers to apply to separated and reattaching flows. In this method, the first moment of the momentum integral across the boundary layer is used in addition to the momentum integral. By replacing Tani's quartic velocity profile with Stewartson's lower branch profiles, better agreement was obtained with Chapman's experimental pressure distribution data for shock induced separation at $M = 2$. For the cooled wall problem, Cohen and Reshotko upper and lower branch similar solutions for heat transfer were used by Savage [28] and it was shown that the length of the separated flow decreases as cooling is increased, which result is qualitatively in agreement with experiment. However, certain anomalies upstream of separation for highly cooled surfaces led Lees to suggest the use of two-parameter velocity profiles in a two-moment integral approach. Using this method Lees and Reeves [27] predict that at a certain ratio of wall to free stream temperature ratio, depending on Mach number, the laminar boundary layer becomes "supercritical". The upstream interaction commences with a "shock" which brings the boundary layer to a subcritical state in a few boundary-layer thicknesses, after which the flow proceeds through separation.

3.3.3 *Pohlhausen-type methods.* Both the Stratford–Gadd and Crocco–Lees methods predict a variation of pressure gradient at separation, disagreeing with Gadd's [14] experimental results, which show little effect of wall temperature on the pressure distribution. In view of this discrepancy and the algebraic complexity of the Crocco–Lees method, Bray *et al.* [11] favour the Pohlhausen type approach. Two alternative methods are used; in method (A) the pressure gradient varies as T_W^{-1}, being considerably larger than observed experimentally, and there is no effect on separation pressure; while according to method (B), heat transfer has a smaller effect on the overall shape of the pressure distribution, the rise to separation being increased by cooling as with the Crocco–Lees method, and is more in accord with experiment. The zero heat-transfer pressure coefficient at separation is 0·94 $(M^2 - 1)^{-\frac{1}{4}}$ $Re_{xs}^{-\frac{1}{4}}$, almost exactly in agreement with Hakkinen's experimental data [29]. It was found that the upstream effect was reduced by cooling and that suction had a qualitatively similar effect.

Makofski [30] has extended the Pohlhausen approach using a fifth-degree polynomial with two undetermined parameters to represent the velocity profile. The computed results for pressure and skin-friction distributions indicate excellent agreement with Hakkinen's experiment at $M = 2$ and show a definite improvement over the single-parameter Pohlhausen analysis, especially at the plateau.

4. HEAT TRANSFER IN SEPARATED AND REATTACHING FLOWS

4.1 *Chapman's mixing model*

Chapman [31] has extended his simple theory, for the dead-air pressure in pure laminar separations with negligible boundary-layer thickness at separation, to include heat transfer. Cases with gas injection and turbulent separation were also considered. The flow field analysed consisted of a thin constant pressure viscous mixing layer separated from a solid surface by an enclosed region of low velocity—the "dead-air" region, Fig. 4. The law of conservation of energy was employed to relate calculated conditions within the separated mixing layer to the rate of heat transfer at the surface. The differential equations, with appropriate boundary conditions were solved subject to $Pr =$ constant, $p/\rho = RT$ and $\mu \propto T$. The ratio of the average heat-transfer coefficient in the laminar mixing zone $\bar{h}_s$, to the attached laminar value $\bar{h}_A$, at corresponding values of Mach number, Reynolds number and wall to free-stream temperature ratio, was found to be a function of Prandtl number only. Typically, for air, $Pr = 0{\cdot}72$, the heat transfer in the separated flow region is predicted to be 56 per cent of the attached flow value. Chung and Viegas [32] have extended Chapman's original analysis for the mixing zone to the reattachment zone, Fig. 4, and predict that in this zone $\bar{h}_s/\bar{h}_A = 4$–5, so that there may well be a net increase in heat transfer in laminar separated flow when reattachment is taken into account. Chapman calculated that a moderate quantity of gas injection reduces the heat transfer in a laminar mixing zone to zero. The analysis can be applied to axially symmetric flow using Mangler's transformation. For heat transfer in separated turbulent flows, Chapman used integral considerations and, assuming $Pr = 1$ and the available data on incompressible and compressible turbulent mixing layers, it was found that the ratio $\bar{h}_s/\bar{h}_A$ is strongly dependent on Mach number and in contrast to the result for laminar layers, a large increase in heat transfer is predicted for turbulent layers when separated at subsonic and low supersonic Mach numbers.

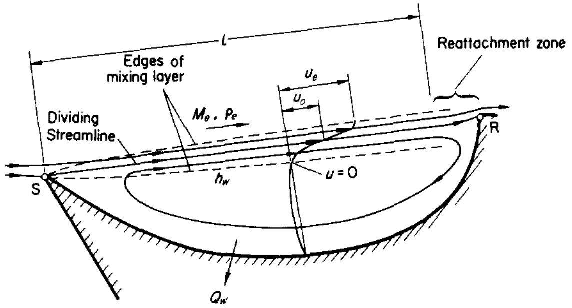


FIG. 4. Chapman's separated flow field (vertical scale expanded) [31].

Larson [16] has compared experimental heat-transfer data for equivalent separated and attached boundary layers with Chapman's predictions. Results were obtained for completely laminar layers in supersonic flow and for turbulent layers over a wide range of subsonic and supersonic Mach numbers, $M = 0{\cdot}3$–$4{\cdot}0$. Measurements of the effect of wall cooling on separated boundary-layer transition were also included. Axially symmetric and two-dimensional models were used, and the local power input to provide constant wall temperature over the surfaces was measured for several wall temperatures greater than the adiabatic. An average heat-transfer coefficient, defined as $\bar{h} = [\mathrm{d}(Q_{\mathrm{tot}}/A)/\mathrm{d}T_W]$, was obtained from Q_{tot}/A vs. T_W, h being found to be independent of T_W.

The results for two and three-dimensional laminar boundary layers up to the Reynolds number at which transition begins indicate that the average heat transfer for laminar layers is reduced in accordance with Chapman's theory and is independent of the Mach number, Reynolds number and the heated surface area A, beneath the separated layer. To obtain turbulent boundary-layer data, boundary-layer strips were fitted to axially symmetric models to provide minimum thickness turbulent layers at separation for various Mach and Reynolds numbers. The ratio of the average heat transfer in separated turbulent flow to the corresponding attached flow value was found to be about 0·6. A large discrepancy between Larson's observations and Chapman's theory is evident, especially at the lower Mach numbers. To clarify this, velocity and temperature profiles were measured in separated turbulent boundary layers with heated and unheated walls. The lower portion of the probe surveys indicated that the measured temperature difference was an order of magnitude less than that assumed by the theory. The surveys also showed that the thickness of the boundary layer was greater than half the depth of the separated region, which may contribute to the difference since the theory assumes that the thickness is relatively small. Larson concludes that the theory requires modification to take into account the details of the reversed-flow region.

Larson's heat-transfer results for separated and attached laminar layers show the same dependence on Reynolds number, i.e. $St \propto Re_L^{-\frac{1}{2}}$ where Stanton number $St = (\bar{h}/\rho U C_p)$. The data for attached turbulent boundary layers exhibit the usual dependence $St \propto Re_L^{-\frac{1}{5}}$, however, for the separated flow data $St \propto Re_L^{-\frac{2}{5}}$, indicating that the reduction in heat transfer by separation increases with Reynolds number and that considerable reductions in average heat transfer are possible through turbulent separation. In all cases the maximum local heat transfer occurred in the reattachment zone of the separated flow, while for attached flows the maximum was measured at the portion of the heated surface nearest the leading edge. This led Larson and others to suggest that multiple separations over a surface may enable the distribution of local heat transfer to be controlled and result in a net reduction in heat transfer, Crawford and Rumsey [33].

Naysmith [34], using more sensitive heat flow meters, has shown the existence of pronounced peaks in the local heat-transfer distribution in the reattachment zone of laminar and turbulent layers behind backward facing steps. Rom and Seginer [35] have measured the heat transfer behind backward facing steps in laminar flow in a shock tube, using platinum film gauges, at $M = 1{\cdot}5$–$2{\cdot}5$ and $Re_L = 2 \times 10^3$–2×10^5, where L is the length of flat plate ahead of the step. They found that the local heat transfer depends on the ratio of boundary-layer thickness at separation to step height, the ratio being represented by the parameter $(L/h)Re_L^{\frac{1}{2}}$. For $(L/h)Re_L^{\frac{1}{2}}$ than 0·067 the local heat transfer increases greater than 0·067 the local heat transfer increases gradually through the reattachment zone, while for relatively thin boundary layers, $(L/h)Re_L^{\frac{1}{2}}$ less than 0·067, there is a sharp peak at reattachment corresponding to an abrupt pressure rise and relatively high local velocity gradients. The average heat transfer for the separated region increases as the boundary-layer thickness is decreased, such that for $(L/h)Re_L^{\frac{1}{2}}$ less than 0·07 the average heat transfer is greater than the attached flow value. In an investigation of laminar hypersonic cavity flows at $M = 11$ Nicoll [36] found that the heat transfer was in agreement with Chapman's theory for the separated region but was only half that estimated by Chung and Viegas for the reattachment zone, the net reduction being about 20 per cent of the attached flow value. Miller *et al.* [37] have studied pressure and heat-transfer distributions in wedge and spoiler induced hypersonic laminar separations in the Mach number range 8–22, with unit Reynolds numbers in the range $0{\cdot}9 \times 10^5$–30×10^5 per foot. Typical results at $M = 16$ show that there is a distinct increase in heat transfer in the reattachment zone. The average heat transfer in the mixing zone was about 60 per cent of Chapman's prediction using plateau pressure conditions, while the heat-transfer distribution in the reattachment zone was similar to the reattachment pressure rise, and could be calculated from the measured pressure distribution. The same general trends in local heat transfer were observed by Needham [38] at $M = 10$. Hypersonic flow separation and aerodynamic control characteristics of basic geometrics with typical control surfaces have also been investigated by Kaufman [39] and Hartofilis [40] who present pressure and heat-transfer measurements for $M = 8$ and $M = 13$ and 19 respectively. For the case of separated flow behind a circular cylinder in subsonic flow, Richardson [41] and Acrivos *et al.* [42] give heat-transfer data for an extensive Reynolds number range.

Gadd *et al.* [15] have obtained local heat-transfer and skin friction measurements for separated and attached turbulent boundary layers at $M = 2{\cdot}44$. The highest rates of heat transfer were found near reattachment just ahead of the forward facing step and downstream of the rearward facing step, the maximum local value being about twice the attached flow value ahead of the step. Similar trends have been observed by Thomman [17] at $M = 1{\cdot}8$ in an investigation covering heat-transfer and skin friction measurements for steps, spoilers and rectangular cavities, in which the maximum local increase in heat transfer was 1·5 times the attached flow value, in the reattachment zone downstream of a spoiler. These experiments effectively extend Larson's investigation [16] to cover the case of two-dimensional separated flow with an initially thick turbulent boundary layer. Although direct comparison with Chapman's analysis [31] is not possible, the results show a remarkably small influence of separation on average heat transfer. Also the divergence between skin friction and heat transfer is not fully explained.

4.2 *Charwat's mass-exchange model*

Charwat *et al.* [18] have investigated the pressure distribution and heat-transfer characteristics of rectangular cavities in turbulent flow at $M = 2$–$3{\cdot}5$. It was found that in supersonic flow a critical length–height ratio of cavity exists below which the flow bridges the cut-out, ("open" cavity), and above which the flow attaches to and separates from the floor of the cut-out in a manner similar to step flows, ("closed" cavity). For turbulent flow in the Mach number range $M = 1{\cdot}5$–4 $(L/H)_{\text{crit.}} \doteqdot 11$, while for subsonic flow the boundaries of the cavity regimes are less clearly defined. The pressure distribution was found to depend on the ratios δ/H and L/H, where δ is the shear layer thickness, L the cavity length and H the cavity height.

The heat-transfer results for open cavities with turbulent upstream boundary layers, indicate an increase in heat-transfer coefficient towards reattachment, in general agreement with Larson' results for relatively thin turbulent boundary layers. However, observations of pulsations within the cavities led to the suggestion that the rate of heat exchange depends partially on an unsteady mass exchange brought about by the fluctuation of the separation streamline. An analysis based on a mass-exchange model led to the result that the mean heat transfer is $St \doteqdot 1/\phi\ \delta_s/L$, where δ_s is the thickness of the shear layer just ahead of recompression, L is the length of the notch and $\phi = [\mathrm{d}(u/u_o)/\mathrm{d}(y/\delta_s)]$ at the mean separating streamline. This implies that the heat transfer is independent of the state of the boundary layer (laminar or or turbulent) and the Mach number, but increases with boundary-layer thickness, as found experimentally. It is found that pulsation is a function of geometry, so that not all cavities pulsate, and it is probable that both the Chapman mixing model and the mass-exchange model share in the heat transfer, the proportion depending on geometry. Charwat points out that the heat transfer to a notched wall per unit drag can be about 5 per cent of that for a flat plate, although the pressure and heat–transfer distributions may be adversely influenced by external pressure gradients.

Furey [43] has investigated the heat-transfer and drag characteristics of various circular cavities at $M = 4{\cdot}5$, for which the boundary layer was laminar at separation, and compared the results with attached flow under the same free-stream conditions. Time averaged velocity profiles and pressure-distribution measurements were made, the results being consistent with an unsteady flow in the cavity, and it was found that the heat-transfer rates were very sensitive to the shape of the recompression face. The heat-transfer results agree with the trend indicated by Charwat when the velocity gradient at the separating streamline is estimated and it is concluded that for open cavities the shape of the recompression surface is crucial in reducing heat transfer to less than that for a flat plate.

4.3 *Three-dimensional effects*

4.3.1 *Three-dimensional perturbations in laminar separation.* Ginoux [44] has investigated the effect of three-dimensional flow perturbations on the heat transfer in the reattachment region of a laminar flow behind a two-dimensional backward facing step at $M \doteqdot 2$. Previous work on free interactions revealed the existence of such perturbations arising from extremely small leading edge irregularities. Pressure surveys indicated constant spanwise static pressure on the reattachment surface, while Pitot-pressure varied periodically along the span, implying a similar variation of skin-friction coefficient and heat-transfer coefficient. It was found that the mean heat-transfer rate in the reattachment region was increased by up to 70 per cent by the presence of strong flow perturbations induced by sticking strips of Scotch-tape around the nose of the model. The recovery factor increased from the laminar value $Pr^{\frac{1}{2}}$ to the turbulent value $Pr^{\frac{1}{3}}$ approximately, there being no indication of transition. The characteristics observed in flow-visualization studies support the view that the perturbations are similar to subsonic Taylor-Görtler vortices on concave walls, the curvature of the flow being assumed sufficient to amplify instability and support the vortex system. Miller *et al* [37] suggest that this effect could also be responsible for a slight increase in heat transfer just before separation.

4.3.2 *Cellular vortices in turbulent separation.* Gadd *et al.* [15] and Thomman [17] present streamline patterns for step and rectangular cavity flows which show the possibility of separation within separated regions. This effect has been observed experimentally by Tani [45] and Abott and Kline [46] in low-speed investigations of turbulent flow over backward facing steps in water channels, using respectively aluminium powder and dye for flow visualization. Abott and Kline found, immediately downstream of the step face, a region of three-dimensional flow characterized by one or more vortices rotating about an axis normal to the main flow direction, Fig. 5. Downstream of this was a two-dimensional reversed flow, with a time dependent tail region, which periodically changed in size. It was also found that near the reattachment region the flow was not two-dimensional. Maull and East [47] have also observed cellular vortex formations in rectangular cavities in a low-speed wind-tunnel wall, using oil flow for visualization. They found that the configuration and steadiness of the cell formation depend on the geometry and correlate with the spanwise pressure distribution on the cavity floor, the non-dimensional cell span being related to the non-dimensional depth as $S/b = 0{\cdot}6\ (1 + d/b)$ for rectangular cavities, where b is the cavity length.

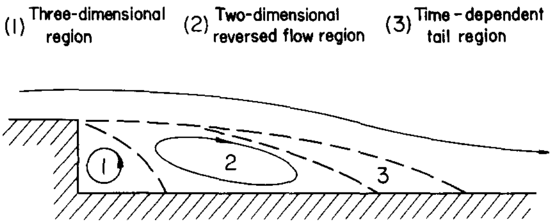


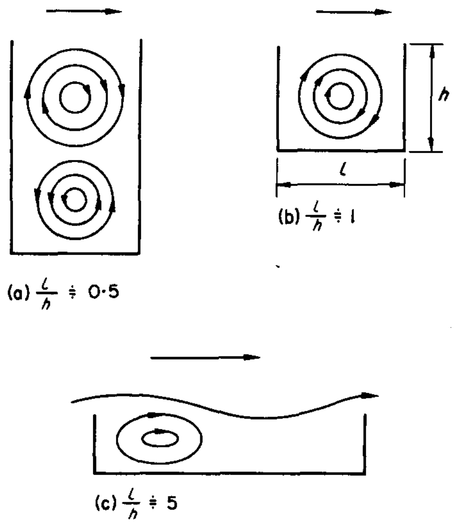
FIG. 5. Flow pattern downstream of a rearward facing step [46].

4.3.3 *Interference effects on two-dimensional models.* Several investigators [6, 10, 17, 18, 37, 48] have observed three-dimensional interference in transonic, supersonic and hypersonic separations, particularly on models with low aspect ratio and in narrow working sections. Typically, interference is induced by leading-edge tip effects and interaction between the sidewall boundary layer and the main separation shock. Bogdonoff and Kepler [6] found it extremely difficult to correlate supposedly two-dimensional data due to three-dimensional effects, while Thomman [17] assumed reasonable two-dimensionality on the centreline, although three-dimensional effects could account for the disagreement with Charwat's heat-transfer results [18]. Charwat *et al.*, using carefully aligned fences, conclude that two-dimensional flow is achieved only under extremely carefully controlled conditions. Roshko and Thomke [48] point out that side plates do not guarantee the elimination of end effects and recommend large, hollow axisymmetric configurations to eliminate the problem. Gray [49] has found that for laminar boundary-layer separation on flared bodies at supersonic speeds there is no practical difference in separation extent between two-dimensional and axisymmetric configurations.

4.4 *Subsonic cavity flows*

In view of the complications of three-dimensional flow and unsteady effects observed by Charwat *et al.* [18] and Abott and Kline [46], Seban [50, 51] and Fox [50, 52] have undertaken examinations of temperature, velocity and heat-transfer distributions in rectangular cavities and in the region downstream of rearward facing steps with turbulent separation. The results of Seban's experiment [51] serve to emphasize the independence of heat-transfer and skin friction. Fox [52] has investigated turbulent flow in various rectangular cavities with length–height ratios of $\frac{1}{4}$–$1\frac{3}{4}$. Mean heat-transfer coefficients were found to be proportional to the free stream mass velocity to the 0·8 power, (at variance with Larson's measurements and Charwat's theory), and for a given length–height ratio, proportional to the cavity size to the $-0{\cdot}2$ power. The maximum local value of heat-transfer coefficient occurred in the reattachment region at the top of the downstream side of the cavity and was roughly proportional to the 0·6 power of the mass velocity. Pressure coefficients were found to have no regular trend with cavity geometry and maximum values of 0·5 approximately were observed in a cavity with a length–height ratio of 1·5.

Several investigations of heat transfer in gas-cooled nuclear reactors have involved experiments on turbulent separated and reattaching flows. Typical of these are studies of heat-transfer and fluid-flow aspects of multiple transverse fins by Harris and Wilson [53] and Ueda and Harada [54] in which pairs of "geared vortices" have been observed in cavities with length–height ratios less than one. Ueda and Harada found that for $1{\cdot}4 \times 10^4 < Re < 20 \times 10^4$ regular flow patterns occur between fins as a function of length–height ratio of the cavity, Fig. 6, the most effective heat transfer to pressure-drop performance being obtained for a length–height ratio of 1·2–1·6. In connection with nuclear fuel elements Emerson and Morris [55] have found that the increase in local heat



(a) $\frac{l}{h} \doteqdot 0{\cdot}5$

(b) $\frac{l}{h} \doteqdot 1$

(c) $\frac{l}{h} \doteqdot 5$

FIG. 6. Flow patterns between transverse fins [54].

transfer resulting from the presence of a small unheated groove in the wall of an electrically heated tube, $25000 < Re < 250000$, was small and approximately the same as that obtained downstream of an unheated length, equal to the width of the groove, in a smooth tube.

Investigations of cavity flows without heat transfer include those of Maull and East [47] and Mills [56], who compares theoretical and experimental velocity profiles in a square cavity.

5. CONCLUSIONS

The results of many investigations have given a clearer picture of the complex, often three-dimensional and unsteady flow, involved in separated and reattaching flows.

Relative to attached flow, distinct heat-transfer characteristics are evident in the mixing and reattachment zones of nominally two-dimensional separated flow. In laminar flow, predictions of a large reduction in heat transfer in the mixing zone [18, 31] and an increase in the reattachment zone [32] are observed experimentally [16, 34, 38, 43] so that care is required to achieve a net reduction in heat transfer. In turbulent flow, the same general trend in heat transfer is evident to a lesser degree [15–18, 50–53], but is not so well predicted [18, 31, 52].

Boundary-layer thickness and surface geometry, especially reattachment curvature, are important parameters in controlling the heat-transfer distribution [18, 35, 43]. In practice the choice of a surface for a given heat-transfer drag ratio and the estimation of the heat transfer for a given geometry are simplified since the separation pressure distribution is virtually independent of heat transfer while for reattachment the heat-transfer distribution tends to follow the pressure rise [11, 15, 18, 35, 37, 38] from which the reattachment heating rise can be forecast [37, 38]. Calculations of the adiabatic pressure distribution at moderate supersonic speeds [11, 25, 27, 30] are in good agreement with experimental results [6, 7, 14, 29].

In view of the effect of three-dimensional perturbations in laminar flow reattachment zone heat transfer [44] and the evidence of unsteady reattachment in turbulent flow [18, 46] further information is required on the local heat transfer in and just downstream of this crucial zone. There is also a need to investigate the conditions for pulsation in subsonic and supersonic cavity flow [18, 52] and to determine the influence of cellular vortex stability on turbulent subsonic cavity flow [47].

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Résumé—Les aspects théoriques et expérimentaux des écoulements décollés et recollés sont étudiés en faisant spécialement attention aux caractéristiques de transport de chaleur de ces écoulements.

Zusammenfassung—Theoretische und experimentelle Gesichtspunkte bei sich ablösenden und wiederanlegenden Strömungen werden unter besonderer Berücksichtigung des Wärmeübergangs behandelt.

Аннотация—Дается обзор экспериментальных и теоретических данных об отделившемся и вновь присоединившемся потоках со специальной ссылкой на теплообменные характеристики таких потоков.